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THE EFFECT OF DIKE INTRUSION ON FREE CONVECTION IN CONDUCTION-DOMINATED GEOTHERMAL RESERVOIRS

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Abstract—A perturbation analysis is made for the study of the effect of dike intrusion on free convection in unconfined conduction-dominated geothermal reservoirs. The perturbation equations are of elliptic type that can be solved numerically by the finite difference method. Up to the second-order approximations are retained in the numerical computation. The effects of dike intrusion on streamlines, temperature distribution, and the shape of water table in a two-dimensional volcanic island aquifer are shown.

NOMENCLATURE

D, $D \equiv \rho_s K g h/\alpha\mu$;
F, function denoting the position of water table;
g, gravity vector;
h, depth of the reservoir at the ocean sides;
k_m, thermal conductivity of the formation;
K, permeability of the formation;
l, the width of the reservoir;
L, the dimensionless width of the reservoir, $L \equiv l/h$;
m, dummy index in equation (10);
$\bar{n}$, unit vector normal to water table;
p, pressure;
P, dimensionless pressure, $P \equiv (p-p_a)/\rho_s g h$;
P_1, P_2, first-order and second-order perturbation functions for pressure;
T, temperature;
T_d, temperature of the dike;
T_L, prescribed temperature of the impermeable surface;
u, v, Darcy's velocity in the x and y directions;
x, y, Cartesian coordinate system;
X, Y, dimensionless coordinates.

Greek symbols

α, equivalent thermal diffusivity, $\alpha \equiv k/(\rho C_p)_f$;
β, thermal expansion coefficient;
ε, perturbation parameter, $\varepsilon \equiv \beta(T_c - T_s)$;
η, the height of water table;
$\bar{\eta}$, dimensionless height of water table, $\bar{\eta} \equiv \eta/h$;
η_1, η_2, first-order and second-order perturbation functions for the height of water table;
θ, dimensionless temperature, $\theta \equiv (T-T_s)/(T_c-T_s)$;
$\theta_0, \theta_1, \theta_2$, zero-order, first-order and second-order perturbation functions for temperature;
θ_d, prescribed dimensionless temperature of the dike;
θ_L, prescribed dimensionless temperature of the impermeable surface;
μ, viscosity of convecting fluid;
ρ, density of convecting fluid;
ψ, stream function;
Ψ, dimensionless stream function, $\Psi \equiv \mu\psi/\rho_s g h K$;
Ψ_1, Ψ_2, first-order and second-order dimensionless stream functions.

Subscripts

a, atmospheric condition;
s, condition in the ocean.

INTRODUCTION

MAGMATIC intrusion occurs frequently in the Earth's crust where there are intense tectonic or volcanic activities. The intruded magma then acts as a heat source which in turn heats the ground-water in the aquifer. The heated ground-water is driven buoyantly upward to the top of the aquifer where it can then be tapped for power generation through drill holes. A qualitative assessment of the capacity and location of geothermal resources can sometimes be made from the observation of temperature anomaly in a rock formation or heat flux anomaly on the Earth's surface. A thorough understanding of heat-transfer characteristics in the Earth's crust thus will aid in a correct interpretation of field data during geophysical exploration.

The intrusive magma may take many different forms or sizes. A sheet-like intrusive body is called a dike or a sill depending upon whether it is perpendicular or parallel to the stratification in the bedded rocks. On the basis of the heat-conduction theory, Horai [1] has recently completed a study to relate surface heat flux to the parameters specifying the intrusion such as magmatic temperature, geometry, and dimensions of the intrusive body. However, recent studies [2–4] suggest that the convection of ground-water plays an important role on the heat-transfer characteristics in geothermal areas. Thus, in the present paper, we shall study the effects of dike intrusion on the temperature distribution in an island aquifer (Fig. 1a) taking into account the movement of ground-water. To simplify the problem, the dike is idealized as a vertical impermeable rectangular obstacle while the island aquifer is idealized as a two-dimensional homogeneous and isotropic porous medium bounded vertically by ocean on the sides, with horizontal impermeable surface at the bottom, and unconfined at the top where the position of water table is not known *a priori* (Fig. 1b). Exact numerical solution of the problem is difficult since the convergence of the solution is very sensitive to the position of the water table [5, 6]. For a conduction-dominated geothermal reservoir, however, a perturbation method can be used to approximate the non-linear problem by a set of linear sub-problem that can be solved by finite difference method [2]. Contours for streamlines and temperature distribution, as well as the amount of the upwelling of water table as a result of dike intrusion for a particular set of parameters are presented.

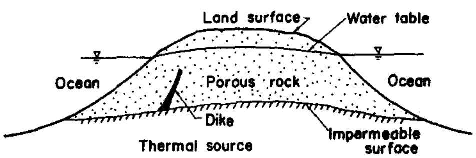

FIG. 1(a). An unconfined aquifer in a volcanic island with dike intrusion.

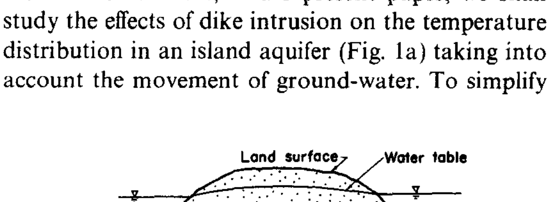

FIG. 1(b). Idealized model of a geothermal reservoir with dike intrusion.

Governing equations and boundary conditions

The governing equations for the simultaneous heat and mass transfer in a porous medium are the continuity equation, Darcy's law, energy equation, and equation of state. To simplify the formulation of the problem, we assume that:

1. The flow field is steady and two-dimensional.
2. There is no rainfall at the water table.
3. The temperature of the fluid, T, is everywhere below boiling for the pressure, p, at that depth.
4. The fluid properties such as specific heat, C, and the kinematic viscosity, μ, as well as the medium properties such as thermal conductivity, k, and permeability, K, are all constant.
5. Density, ρ, is linearly proportional to temperature, i.e. $\rho = \rho_s[1-\beta(T-T_s)]$ where β is the thermal expansion coefficient and the subscript "s" denoting the condition in the ocean.
6. Boussinesq approximation is employed, i.e. density is assumed to be constant except in the bouyancy force term.

With these assumptions it can be shown that the governing equations in terms of dimensionless pressure, P, and temperature, θ, are [2]

$$\frac{\partial^2 P}{\partial X^2}+\frac{\partial^2 P}{\partial Y^2}=\varepsilon\frac{\partial \theta}{\partial Y}, \tag{1}$$

$$\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}+D\left\{\left[\frac{\partial P}{\partial X}\frac{\partial \theta}{\partial X}+\frac{\partial P}{\partial Y}\frac{\partial \theta}{\partial Y}\right]+[1-\varepsilon\theta]\frac{\partial \theta}{\partial Y}\right\}=0, \tag{2}$$

where

$$P \equiv \frac{p-p_a}{\rho_s g h},\quad \theta \equiv \frac{T-T_s}{T_c-T_s},\quad X \equiv \frac{x}{h},\quad L \equiv \frac{l}{h},$$

$$\varepsilon \equiv \beta(T_c-T_s),\quad \text{and}\quad D \equiv \frac{\rho_s K g h}{\alpha \mu}, \tag{3}$$

with h and l denoting the depth and the width of the aquifer, g the gravitational acceleration, and $\alpha \equiv k/(\rho C)_f$ the equivalent thermal diffusivity. The subscript "c" denotes the maximum temperature on the impermeable surface.

The dimensionless boundary conditions along the ocean are

$$P(0,Y) = 1-Y, \tag{4a}$$

$$P(L,Y) = 1-Y, \tag{4b}$$

$$\theta(0,Y) = 0, \tag{4c}$$

$$\theta(L,Y) = 0, \tag{4d}$$

where equations (4a) and (4b) denote hydrostatic pressure and (4c) and (4d) denote constant temperature along the ocean sides.

Along the water table $Y = \bar{\eta}$, the dimensionless boundary conditions are given by

$$P(X,\bar{\eta}) = 0, \tag{5a}$$

$$\theta(X,\bar{\eta}) = \theta_a, \tag{5b}$$

$$\frac{\partial \bar{\eta}}{\partial X}\frac{\partial P}{\partial X}(X,\bar{\eta})-\left(\frac{\partial P}{\partial Y}(X,\bar{\eta})+1-\varepsilon\theta_a\right)=0, \tag{5c}$$

where $\bar{\eta} \equiv \eta/h$, and $\theta_a \equiv (T_a-T_s)/(T_c-T_s)$ with T_a denoting the atmospheric temperature. It is worth mentioning that boundary condition (5c) follows from the conditions $\bar{v}\cdot\bar{n} = 0$ where $\bar{n}$ is the unit vector normal to the water table which is given by $\bar{n} = \nabla F/|\nabla F|$ with $F(X, Y) = Y - \bar{\eta}(X) = 0$ denoting the equation for the position of the water table [2].

If a vertical impermeable dike with uniform temperature T_d is located between X_1 and X_2 on the impermeable surface (Fig. 1b), the dimensionless boundary conditions along the dike are

$$\theta(X_1, Y) = \theta(X_2, Y) = \theta_d, \quad 0 \leqslant Y \leqslant Y_1 \tag{6a}$$

$$\theta(X, Y_1) = \theta_d, \quad X_1 \leqslant X \leqslant X_2 \tag{6b}$$

$$\frac{\partial P}{\partial X}(X_1, Y) = \frac{\partial P}{\partial X}(X_2, Y) = 0, \quad 0 \leqslant Y \leqslant Y_1 \tag{6c}$$

$$\frac{\partial P}{\partial Y}(X, Y_1) = -1 + \varepsilon\theta_d, \quad X_1 \leqslant X \leqslant X_2 \tag{6d}$$

where $\theta_d \equiv (T_d - T_s)/(T_c - T_s)$. It should be noted that equations (6c) and (6d) follow from Darcy's law and the fact that velocity normal to the surface of the dike vanishes. Similarly, if temperature on the rest of the horizontal impermeable surface is prescribed, the boundary conditions are given by

$$\theta(X, 0) = \theta_L(X), \quad 0 \leqslant X \leqslant X_1 \text{ and } X_2 \leqslant X \leqslant L \tag{7a}$$

$$\frac{\partial P}{\partial Y}(X, 0) = -1 + \varepsilon\theta_L(X), \quad 0 \leqslant X \leqslant X_1 \text{ and } X_2 \leqslant X \leqslant L. \tag{7b}$$

Equations (1) and (2) with boundary conditions (4)–(7) are a set of non-linear partial differential equations with non-linear boundary conditions for the determination of pressure and temperature in a hot-water aquifer with a vertical dike.

After θ is obtained, the dimensionless stream function $\psi \equiv \mu\Psi/\rho_s ghK$ can be determined from

$$\frac{\partial^2\psi}{\partial X^2} + \frac{\partial^2\psi}{\partial Y^2} = -\varepsilon\frac{\partial\theta}{\partial X}, \tag{8}$$

where equation (8) is obtained from the elimination of pressure in Darcy's law and from the definition of the stream function, i.e. $u = \partial\Psi/\partial y$ and $v = -\partial\Psi/\partial x$. The boundary conditions for ψ along the ocean are given by

$$\frac{\partial\psi}{\partial X}(0, Y) = \frac{\partial\psi}{\partial X}(L, Y) = 0, \tag{9a}$$

i.e. the vertical velocity is zero along the ocean implying that $\theta_L(X) = 0$ as $X \to 0$ and $X \to L$. Along the water table, the impermeable surface, and on the dike, the boundary condition for ψ is

$$\psi = 0. \tag{9b}$$

Perturbation analysis

If the value of ε in equations (6)–(8) is small, we can obtain a perturbation solution to the problem. For this purpose, we now assume that dependent variables be expanded in a power series of ε. Thus, we have

$$\theta(X, Y) = \sum_{m=0}^{\infty} \varepsilon^m\theta_m(X, Y), \tag{10a}$$

$$P(X, Y) = (1 - Y) + \sum_{m=1}^{\infty} \varepsilon^m P_m(X, Y), \tag{10b}$$

$$\psi(X, Y) = \sum_{m=1}^{\infty} \varepsilon^m\psi_m(X, Y), \tag{10c}$$

$$\bar{\eta}(X) = 1 + \sum_{m=1}^{\infty} \varepsilon^m\eta_m(X), \tag{10d}$$

where $P_m(X, Y)$, $\theta_m(X, Y)$, $\psi_m(X, Y)$ and $\eta_m(X)$ are perturbation functions to be determined. Substituting equation (10) into equations (1)–(9), making a Taylor's series expansion on boundary conditions (5) and collecting terms of like power in ε, we have the following set of subproblems.

Zero-order approximations

The zero-order problem for θ is given by

$$\frac{\partial^2\theta_0}{\partial X^2} + \frac{\partial^2\theta_0}{\partial Y^2} = 0, \tag{11}$$

with boundary conditions given by

$$\theta_0(0, Y) = \theta_0(L, Y) = 0, \tag{12a}$$

$$\theta_0(X, 1) = \theta_a, \tag{12b}$$

$$\theta_0(X, 0) = \theta_L(X), \tag{12c}$$

$$\theta_0(X_1, Y) = \theta_0(X_2, Y) = \theta_d, \quad \text{for} \quad 0 \leqslant Y \leqslant Y_1, \tag{12d}$$

$$\theta_0(X, Y_1) = \theta_d, \quad \text{for} \quad X_2 \leqslant X \leqslant X_1. \tag{12e}$$

First-order approximations

The first-order problem for P is given by

$$\frac{\partial^2 P_1}{\partial X^2} + \frac{\partial^2 P_1}{\partial Y^2} = \frac{\partial\theta_0}{\partial Y}, \tag{13}$$

where the RHS of equation (13) is known from the zero-order problem. The boundary conditions for P_1 are given by

$$P_1(0, Y) = P_1(L, Y) = 0, \tag{14a}$$

$$\frac{\partial P_1}{\partial Y}(X, 1) = \theta_a, \tag{14b}$$

$$\frac{\partial P_1}{\partial Y}(X, 0) = \theta_L(X), \tag{14c}$$

$$\frac{\partial P_1}{\partial X}(X_1, Y) = \frac{\partial P_1}{\partial X}(X_2, Y) = 0, \quad \text{for} \quad 0 \leqslant Y \leqslant Y_1 \tag{14d}$$

$$\frac{\partial P_1}{\partial Y}(X, Y_1) = \theta_d, \quad \text{for} \quad X_1 \leqslant X \leqslant X_2. \tag{14e}$$

Once P_1 is determined, $\eta_1(X)$ is obtained from

$$\eta_1(X) = P_1(X, 1). \tag{15}$$

which follows from equation (5a).

The first-order approximation for θ is

$$\frac{\partial^2\theta_1}{\partial X^2} + \frac{\partial^2\theta_1}{\partial Y^2} = -D\left[\frac{\partial P_1}{\partial X}\frac{\partial\theta_0}{\partial X} + \frac{\partial P_1}{\partial Y}\frac{\partial\theta_0}{\partial Y} - \theta_0\frac{\partial\theta_0}{\partial Y}\right], \tag{16}$$

with boundary conditions given by

$$\theta_1(0, Y) = \theta_1(L, Y) = 0, \tag{17a}$$

$$\theta_1(X, 0) = 0, \tag{17b}$$

$$\theta_1(X, 1) = -P_1(X, 1)\frac{\partial \theta_0}{\partial Y}(X, 1), \tag{17c}$$

$$\theta_1(X_1, Y) = \theta_1(X_2, Y) = 0, \quad \text{for} \quad 0 \leqslant Y \leqslant Y_1, \tag{17d}$$

$$\theta_1(X, Y) = 0, \quad \text{for} \quad X_1 \leqslant X \leqslant X_2. \tag{17e}$$

The first-order approximation for ψ is given by

$$\frac{\partial^2 \psi_1}{\partial X^2} + \frac{\partial^2 \psi_1}{\partial Y^2} = -\frac{\partial \theta_0}{\partial X_1}. \tag{18}$$

with boundary conditions given by

$$\frac{\partial \psi_1}{\partial X}(0, Y) = \frac{\partial \psi_1}{\partial X}(L, Y) = 0, \tag{19a}$$

$$\psi_1(x, 1) = \psi_1(x, 0) = 0, \tag{19b}$$

and $\psi_1 = 0$ along the surface of the dike.

Second-order approximations

Second-order approximation for pressure is given by

$$\frac{\partial^2 P_2}{\partial X^2} + \frac{\partial^2 P_2}{\partial Y^2} = \frac{\partial \theta_1}{\partial Y}, \tag{20}$$

with boundary conditions given by

$$P_2(0, Y) = P_2(L, Y) = 0, \tag{21a}$$

$$\frac{\partial P_2}{\partial Y}(X, 0) = 0, \tag{21b}$$

$$\frac{\partial P_2}{\partial Y}(X, 1) = \left[\frac{\partial P_1}{\partial X}(X, 1)\right]^2 - \frac{\partial^2 P_1}{\partial Y^2}(X, 1)P_1(X, 1). \tag{21c}$$

$$\frac{\partial P_2}{\partial X}(X_1, Y) = \frac{\partial P_2}{\partial X}(X_2, Y) = 0, \quad 0 \leqslant Y \leqslant Y_1, \tag{21d}$$

$$\frac{\partial P_2}{\partial Y}(X, Y_1) = 0, \quad X_1 \leqslant X \leqslant X_2. \tag{21e}$$

The second-order approximation for temperature is given by

$$\frac{\partial^2 \theta_2}{\partial X^2} + \frac{\partial^2 \theta_2}{\partial Y^2} = -D\left[\frac{\partial P_1}{\partial X}\frac{\partial \theta_1}{\partial X} + \frac{\partial P_1}{\partial Y}\frac{\partial \theta_1}{\partial Y} + \frac{\partial P_2}{\partial X}\frac{\partial \theta_0}{\partial X} + \frac{\partial P_2}{\partial Y}\frac{\partial \theta_0}{\partial Y} - \theta_0\frac{\partial \theta_1}{\partial Y}\right], \tag{22}$$

with boundary conditions given by

$$\theta_2(0, Y) = \theta_2(L, Y) = 0, \tag{23a}$$

$$\theta_2(X, 0) = 0, \tag{23b}$$

$$\theta_2(X, 1) = -\eta_1\frac{\partial \theta_1}{\partial Y}(X, 1) - \eta_2\frac{\partial \theta_0}{\partial Y}(X, 1) - \eta_1^2\frac{\partial^2 \theta_0}{\partial Y^2}(X, 1), \tag{23c}$$

$$\theta_2(X_1, Y) = \theta_2(X_2, Y) = 0, \quad 0 \leqslant Y \leqslant Y_1, \tag{23d}$$

$$\theta_2(X, Y_1) = 0, \quad X_1 \leqslant X \leqslant X_2 \tag{23e}$$

where η_1 is given by equation (15) and

$$\eta_2 = P_2(X, 1) + P_1(X, 1)\frac{\partial P_1}{\partial Y}(X, 1). \tag{23f}$$

The second-order approximation for ψ_2 is given by

$$\frac{\partial^2 \psi_2}{\partial X^2} + \frac{\partial^2 \psi_2}{\partial Y^2} = -\frac{\partial \theta_1}{\partial X}, \tag{24}$$

with boundary conditions given by

$$\frac{\partial \psi_2}{\partial X}(L, Y) = \frac{\partial \psi_2}{\partial X}(0, Y) = 0, \tag{25a}$$

$$\psi_2(X, 0) = 0, \tag{25b}$$

$$\psi_2(X, 1) = -\eta_1\frac{\partial \psi_1}{\partial Y}(X, 1), \tag{25c}$$

$$\psi_2(X_1, Y) = \psi_2(X_2, Y) = 0, \quad 0 \leqslant Y \leqslant Y_1 \tag{25d}$$

$$\psi_2(X, Y_1) = 0, \quad X_1 \leqslant X \leqslant X_2. \tag{25e}$$

The governing equations for the zero-, first- and second-order problems as given by equations (11), (13), (16), (18), (20), (22) and (24) are either the Laplace equation or Poisson equation with nonhomogeneous boundary conditions, which could have been solved in closed form by a separation of variables. Since the numerical evaluation of the resultant expressions in terms of many double and triple Fourier series will be very costly, we therefore resort to the numerical solution of the problem by the finite difference method.

NUMERICAL COMPUTATION AND RESULTS

The Laplace operators in equations (11), (13), (16), (18), (20), (22) and (24) are approximated by the standard five-point formula of the finite difference method, while the derivatives in the boundary conditions for these equations are approximated by the central difference scheme. Computations begin with the determination of θ_0, and in the order of P_1, θ_1, ψ_1, P_2, θ_2 and ψ_2 so that the derivatives in the nonhomogeneous terms of the equations for each subproblem is solved numerically by the Gauss–Seidel iteration method. Computations were carried out up to the second-order approximation for $D = 500$ (upper bound for which the perturbation method is valid) with $L = 4$, $\varepsilon = 0.1$ and $\theta_a = 0.02$ for the following three cases with different prescribed temperatures of θ_d and θ_L.

Case A: Heating from a hot dike

For the problem of geothermal heating due to a hot dike 0.5 unit in height and 2 units in width located at the center of the aquifer with a cold impermeable surface at the bottom, the prescribed temperatures are

$$\theta_d = 1, \quad 1.9 \leqslant X \leqslant 2.1 \quad \text{and} \quad 0 \leqslant Y \leqslant 0.5$$

$$\theta_L(X) = 0, \quad 0 \leqslant X \leqslant 1.9 \quad \text{and} \quad 2.1 \leqslant X \leqslant 4.$$

Case B: Heating from below

For comparison, computations were also carried out for a geothermal reservoir without a dike, but with

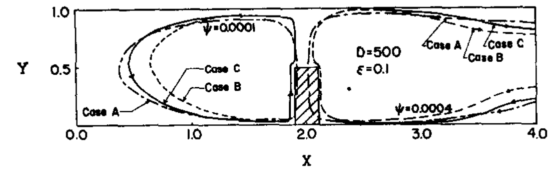

FIG. 2. Effects of vertical and horizontal heating on streamlines in an unconfined geothermal reservoir.

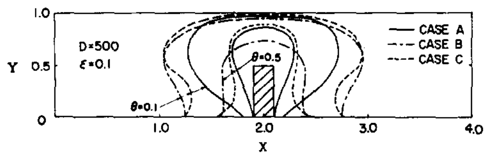

FIG. 3. Effects of vertical and horizontal heating on temperature contours in an unconfined geothermal reservoir.

geothermal heating from the bottom impermeable surface having the following prescribed temperature

$$\theta_L(X) = \exp\left[-\left(\frac{X-2}{0.5}\right)^2\right].$$

Case C: Combined heating

Heating in this case is due to the combination of a vertical dike located at the center of the reservoir as in Case A, and the hot impermeable surface as in Case B. The prescribed temperatures for the heat sources are

$$\theta_d = 1, \quad 1.9 \leqslant X \leqslant 2.1 \quad \text{and} \quad 0 \leqslant Y \leqslant 0.5$$

$$\theta_L(X) = \exp\left[-\left(\frac{X-2}{0.5}\right)^2\right], \quad 0 \leqslant X \leqslant 1.9 \quad \text{and} \quad 2.1 \leqslant X \leqslant 4.0.$$

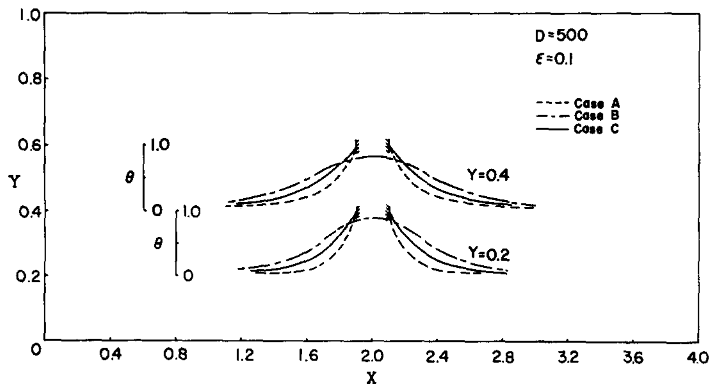

FIG. 4. Horizontal temperature distribution for Cases A, B and C.

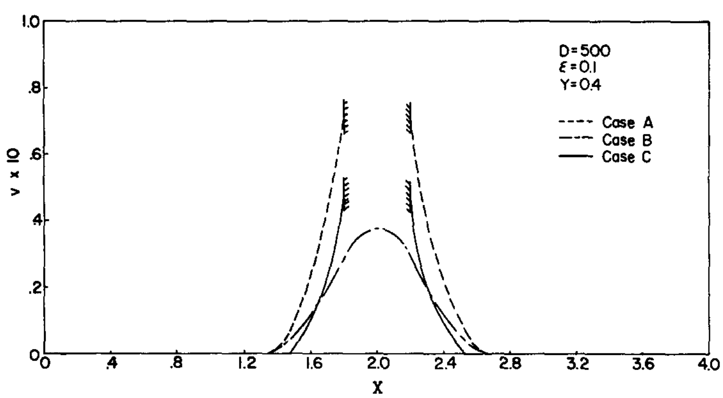

FIG. 5. Vertical velocity profiles at $Y = 0.4$ for Cases A, B and C.

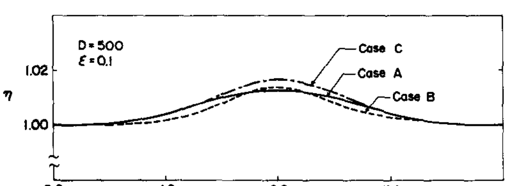

FIG. 6. Effects of heat sources on the upwelling of water table for Cases A, B and C.

Results of Cases A, Band C are plotted in Figs. 2–6. Both the flow pattern and temperature contours are symmetric with respect to $X = 2$. For clarity, however, only $\psi = 0.001$ and $\psi = 0.0004$ are plotted in either side of the aquifer as shown in Fig. 2. The streamlines for the three cases exhibit similar behavior with cold water moving inland in the lower portion of the island aquifer and warm water discharging into the ocean in the upper portion of the aquifer. Near the heat sources, a column of hot fluid rises rapidly which induces two convective cells on either side of the heat sources. When a hot dike exists in the reservoir, the heat source becomes relatively close to the water table. This, plus the fact that the dike provides a larger area for heat transfer, makes it a possibility that hot water can be found at shallow depths, as is shown in Fig. 3. The horizontal temperature distributions at $Y = 0.2$ and $Y = 0.4$ for Cases A, B and C are plotted in Fig. 4, where it is shown that the rate of change in temperature is rapid at the region near the heat source. This boundary layer behavior in temperature distribution is most pronounced for Case A. It is of interest to note that the vertical velocity distribution (Fig. 5) and the temperature distribution (Fig. 4) are similar in shape, and that "velocity slip" occurs on the impermeable surface. The effects of vertical and horizontal heating on the amount of upwelling of water table are shown in Fig. 6—it can be seen here that upwelling of water table increases if a hot dike exists.

CONCLUDING REMARKS

The present perturbation method is valid only for small ε as well as for small and moderate values of D which is applicable to conduction-dominated reservoirs. For large values of D, the solution will break down in the region near the heat source as well as directly above the heat source where convection is predominant.

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L'EFFET DE L'INTRUSION DE DIGUE SUR LA CONVECTION NATURELLE DANS LES RESERVOIRS GEOTHERMIQUES DOMINES PAR LA CONDUCTION

Résumé—Une analyse de perturbation est conduite pour le transfert simultané de chaleur et de masse dans les réservoirs géothermiques non confinés, dominés par la conduction et avec une intrusion de digue. Les équations aux perturbations sont du type elliptique et elles peuvent être résolues numériquement par la méthode aux différences finies. Dans le calcul numérique, les approximations du second ordre sont retenues. On montre les effets de l'intrusion de digue sur les lignes de courant, sur la distribution de température et sur la forme de la table d'eau dans les nappes aquifères bidimensionnelles.

DER EINFLUSS DER DEICH-INTRUSION AUF DIE FREIE KONVEKTION IN LEITUNGSDOMINIERTEN GEOTHERMISCHEN RESERVOIRS

Zusammenfassung—Der gleichzeitige Wärme- und Stoffübergang in unbegrenzten, leitungsdominierten geothermischen Reservoirs bei Deich-Intrusion wird mit Hilfe der Strömungs-Analyse bestimmt. Die Gleichungen sind vom elliptischen Typ und können nach der Methode finiter Differenzen gelöst werden. Dabei werden Näherungen bis zu zweiter Ordnung erhalten. Die Einflüsse der Deich-Intrusion auf den Verlauf von Stromlinien und Temperaturen sowie auf die Form des Wasserspiegels in zwei-dimensionalen Aquifer-Speichern werden gezeigt.

ВЛИЯНИЕ ДАЙКОВОЙ ИНТРУЗИИ НА СВОБОДНУЮ КОНВЕКЦИЮ В ГЕОТЕРМАЛЬНЫХ РЕЗЕРВУАРАХ В УСЛОВИЯХ ПЕРЕДАЧИ ТЕПЛА ТЕПЛОПРОВОДНОСТЬЮ

Аннотация — Методом возмущений анализируется процесс одновременного переноса тепла и массы в неограниченных геотермальных резервуарах с дайковой интрузией в условиях передачи тепла теплопроводностью. Уравнения для возмущений записаны в эллиптическом виде и могут быть решены численно с помощью метода конечных разностей. Численная схема имела второй порядок аппроксимации. Описано влияние дайковой интрузии на линии тока, температурное распределение и форму водной поверхности для двухмерных водоносных пластов.